MENIIT
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JEE MAINS-2017

09-04-2017 (Online)

IMPORTANT INSTRUCTIONS

1. The test is of **3** hours duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360**.
3. There are **three** parts in the question paper A, B, C consisting of **Mathematics, Physics & Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
4. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

PART – A – MATHEMATICS

1. The function $f : N \to N$ defined by $f(x) = x - 5\left[\frac{x}{5}\right]$, where N is the set of natural numbers and [x] denotes the greatest integer less than or equal to x, is

(A) one-one and onto.
(B) one-one but not onto.
(C) onto but not one-one.
(D*) neither one-one nor onto.

Sol. $\left.\begin{array}{l} f(1) = 1 - 5(1/5) = 1 \\ f(6) = 6 - 6[6/5] = 1 \end{array}\right\} \to$ many one

f(10) = 10 – 5(2) = 0 which is not in coduman

many one + into

2. The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is

(A) 16
(B) 14
(C*) – 4
(D) – 5

Sol. $(x - 5)(x^2 + 5x - 50) = 0$

$\Rightarrow (x - 5)(x + 10) (x - 5) = 0$

$\Rightarrow x = 1,5, - 10$ sum = – 4

3. The equation $\text{Im}\left(\frac{iz-2}{z-i}\right) + 1 = 0$, $z \in C$, $z \neq i$ represents a part of a circle having radius equal to

(A) 2
(B) 1
(C*) $\frac{3}{4}$
(D) $\frac{1}{2}$

Sol. Let = x +y

$$\text{Im}\left[\left(\frac{x-y-2}{x+(y-1)}\right)\left(\frac{x-(y-1)}{x+(y-1)}\right)\right] + 1 = 0$$

$$+\frac{(y-1)(y+2)+x^2}{x^2+(y-1)^2} + 1 = 0$$

$= 2x^2 + 2y^2 - y - 1 = 0$

$= x^2 + y^2 - 1/2y - 1/2 = 0$

cot x = 10, 1/4

$= \sqrt{1/2 + 1/2} = \sqrt{9/16} = 3/4$

4. For two 3 × 3 matrices A and B, let A + B = 2B' and 3A + 2B = I_3, where B' is the transpose of B and I_3 is 3 × 3 identity matrix. Then

(A) $5A + 10B = 2 I_3$
(B*) $10A + 5B = 3 I_3$
(C) $B + 2A = I_3$
(D) $3A + 6B = 2 I_3$

Sol. $A^T + B^T = 2B$

$\Rightarrow B = \frac{A^T + B^T}{2}$

$$= A + \left(\frac{B^T + A^T}{2}\right) = 2B^T$$

$2A + A^T = 2B^T$

$$\Rightarrow A = \frac{3B^T - A^T}{2}$$

$3A + 2B = I_3$ ...(i)

$$\Rightarrow 3\left(\frac{3B^T - A^T}{2}\right) + 2\left(\frac{A^T + B^T}{2}\right) = I_3$$

$$\Rightarrow \left(\frac{3B^T + 2B^T}{2}\right) + \left(\frac{2A^T - 3A^T}{2}\right) = I_3$$

$\Rightarrow 11B^T - A^T = 2I_3$ ...(ii)

Equation (i) + (ii)

$35B = 7I_3$

$$\Rightarrow B = \frac{I_3}{5}$$

$$11\frac{I_3}{5} - A = 2I_3$$

$$\Rightarrow 11\frac{I_3}{5} - 2I_3 = A$$

$$\Rightarrow A = \frac{I_3}{5}$$

$\because 5A = 5B = I_3$

$\Rightarrow 10A + 5B = 3I_3$

5. If x = a, y = b, z = c is a solution of the system of linear equations

$x + 8y + 7z = 0$

$9x + 2y + 3z = 0$

$x + y + z = 0$

such that the point (a, b, c) lies on the plane x + 2y + z = 6, then 2a + b + c equals

(A) – 1 (B) 0 (C*) 1 (D) 2

Sol.

$$\left.\begin{aligned} x + 8y + 7z &= 0 \\ 9x + 2y + 3z &= 0 \\ x + y + z &= 0 \end{aligned}\right\} \begin{aligned} &7y + 6z = 0 \\ \\ &7x + z = 0 \end{aligned}$$

$$x = \lambda \quad \left| y = \frac{-6(-7\lambda)}{7} \right| \quad z = -7\lambda$$

$\boxed{x=\lambda}$ $\boxed{y=6\lambda}$ $\boxed{z=-7\lambda}$

$\lambda+12\lambda-7\lambda=6$ | $2\lambda+6\lambda-7\lambda$

$6\lambda=6$ | $=2\lambda$

$\boxed{\lambda=1}$ | $=\boxed{2}$

6. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is

(A*) 5 × 6! (B) 6 × 6! (C) 7! (D) 5×7!

Sol. boy and 2 girls in circle

$5! \times {}^6c_2 \times 2!$

$5! \times \frac{6!}{4!2!} \times 2!$

5×6!

7. The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$, is

(A*) 1 (B) 4 (C) – 4 (D) – 1

Sol. $\left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)}-\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{-10}$

$= (x^{1/3} + 1 - 1 - 1/x^{1/2})^{10}$

$= (x^{1/3} - 1/x^{1/2})$ $r = 1/3, b = 1/2$

$r=\frac{\frac{10}{3}-(-5)}{1/3+\frac{1}{2}}$

$r=\frac{25/3}{(5\frac{1}{2})}=10$

cos. $= 10c_{10}\ (1)\ (-1)^{10} = 1$

8. If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is

(A*) 2 (B) $4^{1/3}$ (C) $4^{2/3}$ (D) 4

Sol. a + c = 2b

$a.c.\left(\frac{a+c}{2}\right)=8$

9. Let $S_n=\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+..........+\frac{1+2+.....+n}{1^3+2^3+......+n^3}$. If 100 S_n = n, then n is equal to

(A*) 199 (B) 99 (C) 200 (D) 19

Sol. $T_n = \dfrac{\dfrac{n+(n+1)}{2}}{\left(\dfrac{n+(n+1)}{2}\right)^2}$

$T_n = \dfrac{2}{n(n+1)}$

$S_n = 2\sum_{n=1}^{n}\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

$= 2\begin{cases} 1 - \dfrac{1}{2} \\ \dfrac{1}{2} - \dfrac{1}{3} \end{cases}$

$\left.\dfrac{1}{n} - \dfrac{1}{n+1}\right\}$

$= 2\left\{1 - \dfrac{1}{n+1}\right\}$

$\boxed{S_n = \dfrac{2n}{n+1}}$

$100 \times \dfrac{2n}{n+1} = n$

n + 1 = 200

n = 199

10. The value of k for which the function $f(x) = \begin{cases} \left(\dfrac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0 < x < \dfrac{\pi}{2} \\ k + \dfrac{2}{5}, & x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$, is

(A) $\dfrac{17}{20}$ (B) $\dfrac{2}{5}$ (C*) $\dfrac{3}{5}$ (D) $\dfrac{-2}{5}$

Sol. k + 2 /5 = (4 /5)

$= k + \dfrac{2}{4} = 1$

$K = \dfrac{3}{5}$

11. If $2x = y^{\frac{1}{5}} + y^{\frac{-1}{5}}$ and $(x^2 - 1)\dfrac{d^2y}{dx^2} + \lambda x \dfrac{dy}{dx} + ky = 0$, then λ + k is equal to

(A) – 23 (B*) – 24 (C) 26 (D) – 26

Sol. $y^{1/5} + y^{-1/5} = 2x$

$\left(\dfrac{1}{5}y^{-4/5} - \dfrac{1}{5}y^{-6/5}\right)\dfrac{dy}{dx} = 2$

$y'(y^{1/5} - y^{-1/5}) = 10y$

$y'\left(2\sqrt{x^2-1}\right) = 10y$

$y''\left(2\sqrt{x^2-1}\right) + y'2\frac{2x}{2\sqrt{x^2-1}} = \sqrt{y'}$

$y''(x^2-1) + xy' = 5\sqrt{x^2-1}(y')$

$\boxed{y''(x^2-1) + xy' - 25 = 0}$

$\lambda = 1,\ k = -25$

12. The function f defined by $f(x) = x^3 - 3x^2 + 5x + 7$, is

(A*) increasing in R

(B) decreasing in R

(C) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$

(D) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$

Sol. $f(x) = x^3 - 3x^2 + 5x + 7$

$f'(x) = 3x^2 - 6x + 5 > 0$

$f'(x) = 3x^2 - 6x + 5 < 0$

$x \in \phi$

13. Let f be a polynomial function such that $f(3x) = f'(x) \cdot f''(x)$, for all $x \in$ R. Then

(A) $f(2) + f'(2) = 28$ (B*) $f''(2) - f'(2) = 0$ (C) $f''(2) - f(2) = 4$ (D) $f(2) - f'(2) + f''(2) = 10$

Sol. $f(x) = ax^3 + bx^2 + cx + \ell$

$f(3x) = 27ax^3 + 9bx^2 + 3cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f(3x) = f'(x) f''(x)$

$27a = 18a^2$

$a = \frac{3}{2},\ b = 0,\ c = 0,\ d = 0$

$f(x) = \frac{3}{2}x^3$

$f'(x) = \frac{9}{2}x^2, f''(x) = 9x$

14. If $f\left(\frac{3x-4}{3x+4}\right) = x + 2$, $x \neq \frac{-4}{3}$, and $\int f(x)dx = A \log|1 - x| + Bx + C$, then the ordered pair (A, B) is equal to : (where C is a constant of integration)

(A) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (B*) $\left(\frac{-8}{3}, \frac{2}{3}\right)$ (C) $\left(\frac{-8}{3}, \frac{-2}{3}\right)$ (D) $\left(\frac{8}{3}, \frac{-2}{3}\right)$

Sol. $f\left(\frac{3x-4}{3x+4}\right) = x+2, x \neq \frac{4}{3}$

Let $\frac{3x-4}{3x+4} = t$

3x – 4 = 3tx + 4t

$x = \frac{4t-4}{3-3t} + 2$

$f(t) = \frac{10-2t}{3-3t}$

$f(x) = \frac{2x-10}{3x-3}$

$\int f(x)dx = \int \frac{2x-10}{3x-3}dx$

$= \int \frac{2x}{3x-3}dx - 10\int \frac{dx}{3x-3}$

$= \frac{2}{3}\int \frac{x-1}{x-1}dx + \frac{2}{3}\int \frac{dx}{x-1} - \frac{10}{3}\int \frac{dx}{x-1} = \frac{2x}{3} - \frac{8}{3}\ln(x-1) + C$

15. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{3/2}} = \frac{F}{k+5}$, then k is equal to

(A*) 1 (B) 2 (C) 3 (D) 4

Sol. $\int_1^2 \frac{dx}{((x-1)+3)^{3/2}}$

$x - 1 = \sqrt{3}\tan Q$

$= \sqrt{3}\sec^2 Q$

$\int_0^{\pi/6} \frac{\sqrt{3}\sec dQ}{3\sqrt{3}\sec.3Q}$

$= \frac{1}{3}\int_0^{\pi/6} \operatorname{cosec} Q = \frac{1}{3}(\tan Q)_0^{\pi/6}$

$= \frac{1}{6} = \frac{k}{k+5} = k+5 = 6k$

$= \boxed{k=1}$

16. If $\lim\limits_{n\to\infty} \dfrac{1^a + 2^a + ... + n^a}{(n+1)^{a-1}\left[(na+1)+(na+2)+......+(na+n)\right]} = \dfrac{1}{6}$ for some positive real number a, then a is equal to

(A*) 7 (B) 8 (C) $\dfrac{15}{2}$ (D) $\dfrac{17}{2}$

Sol. $\lim\limits_{x\to\infty} \dfrac{\dfrac{1}{(a+1)} \cdot n^{a+1} + a_1 n^a + a_2 n^{a-1} + ...}{(n+1)^{a-1}.n^2 \left(a + \dfrac{1+\frac{1}{n}}{2}\right)} = \dfrac{1}{60}$

$\Rightarrow \quad \lim\limits_{x\to\infty} \dfrac{\dfrac{1}{a+1} + \dfrac{a_1}{n} + \dfrac{a_2}{n} + ...}{\left(1+\dfrac{1}{n}\right)^a \left(a + \dfrac{1+\frac{1}{n}}{2}\right)} = \dfrac{1}{60}$

$\Rightarrow \dfrac{\dfrac{1}{a+1}}{\left(a+\dfrac{1}{2}\right)} = \dfrac{1}{60} \Rightarrow (a+1)(2a+1) = 120$

$2a^2 + 3a - 119 = 0$

$2a^2 + 17a - 14a - 119 = 0$

$\Rightarrow \quad (a - 7)(2a + 17) = 0$

$a = 7, \ -\dfrac{17}{2}$

17. A tangent to the curve, y = f (x) at P(x, y) meets x-axis at A and y-axis at B. If AP : BP = 1 : 3 and f (1)=1, then the curve also passes through the point

(A) $\left(\dfrac{1}{3}, 24\right)$ (B) $\left(\dfrac{1}{2}, 4\right)$ (C*) $\left(2, \dfrac{1}{8}\right)$ (D) $\left(3, \dfrac{1}{28}\right)$

18. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is

(A) $2\sqrt{3} - 1$ (B*) $2\sqrt{3} - 2$ (C) $\sqrt{3} - 2$ (D) $\sqrt{3} - 1$

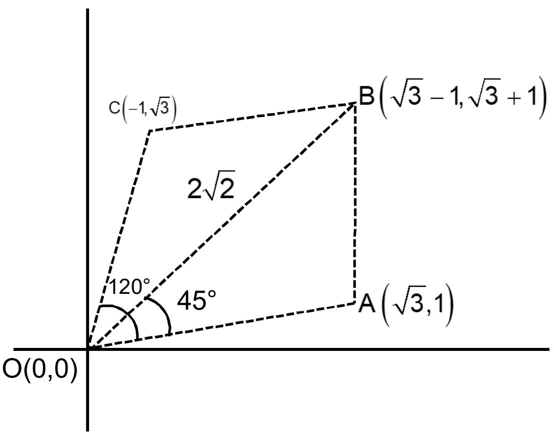
Sol. $\dfrac{x}{\cos 30°} = \dfrac{y}{\sin 30°} = 2$

$x = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$

y = 1

$\dfrac{x}{\cos 120°} = \dfrac{y}{\sin 120°} = 2$

$x = -1, y = \sqrt{3}$



$$\frac{x}{\cos 75^\circ} = \frac{y}{\sin 75^\circ} = 2\sqrt{2}$$

$x = \sqrt{3} - 1$

$y = \sqrt{3} + 1$

$\text{sum} = 0 + \sqrt{3} + \sqrt{3} - 1 + (-1)$

$= 2\sqrt{3} - 2$

19. A line drawn through the point P(4, 7) cuts the circle $x^2 + y^2 = 9$ at the points A and B. Then PA·PB is equal to

(A) 53 (B*) 56 (C) 74 (D) 65

Sol. P(4,7) it midpoint the circle

$\text{PA PB} = S_1^2 = PT^2$

$S_1 = \sqrt{16 + 49 - 9} = \sqrt{56}$

$S_1^2 = 56$; PA –PB = 56

20. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, – 1) and (– 2, 2) is

(A) $\frac{1}{2}$ (B) $\frac{2}{\sqrt{5}}$ (C*) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$

Sol. e = ?, centre at (0,0)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{16}{a^2} - \frac{1}{b^2} = 1$

$16b^2 + a^2 = a^2b^2$ (1)

$\frac{4}{a^2} + \frac{4}{b^2} = 1$

$4b^2 + 4a^2 = a^2b^2$ (2)

From (1) & (2)

$16b^2 + a^2 = 4a^2 + 4b^2$

$3a^2 = 12b^2 = \boxed{a^2 = 4b^2}$

21. If y = mx + c is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then | c | is equal to

(A) $2\sqrt{3}$ (B) $8\sqrt{3}$ (C*) $10\sqrt{3}$ (D) $16\sqrt{3}$

Sol. $c = -29m - 9m^3$

a = 2

Given $(at^2 - a)^2 + 4a^2t^2 = 64$

$\rightarrow (a(t^2 + 1)) = 8$

$\Rightarrow t^2 + 1 = 4 = t^2 = 3$

$\Rightarrow t = \sqrt{3}$

$\therefore$ C = – a [–2t – t^3] = $2at(2 + t^2)$

$= 2\sqrt{3}(5)$

$|c| = 10\sqrt{3}$

22. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is

(A*) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$ (B) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$

(C) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$ (D) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

Sol. Let Centroid be (h,k,ℓ)

$\therefore$ x – intp = 3h Y– intp = 3k, 3–int = 3ℓ

Equ. $\frac{x}{3h}+\frac{y}{3k}+\frac{Z}{3\ell}=1$

dist from (0,0,0)

$$\left|\frac{-1}{\sqrt{\frac{1}{9h^2}+\frac{1}{9k^2}+\frac{1}{9\ell^2}}}\right| = 3$$

$$\boxed{\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1}$$

23. If the line $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x - 4y + 3z = 2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is

(A) 2 (B) 1 (C*) 0 (D) 3

Sol. pt $(3, -2, \lambda)$ on pline $2x - 4y + 3z - 2 = 0$

$= 6 + 8 - 3\lambda - 2 = 0$

$= 3\lambda = 12$ $\boxed{\lambda = 4}$

Now

$\frac{x-3}{1}=\frac{y-2}{-1}=\frac{z+4}{-2}=k_1$ ...(1)

$\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}=k_2$ ...(2)

pt on give 1 = $(k_1+3, -k_1-2, -2k_1-4)$

pt on give 2= $(12k_2+1, 9k_2, 4k_2)$

$k_1+3=12k_2+1 \mid -k_1-2=9k_2 \mid -2k_1-4=4k_2$

$\left.\begin{matrix} k_2=0 \\ k_1=-2 \end{matrix}\right]$

p (1,0,0)

gives are ditersech – thortest distance = 0

24. If the vectors $\vec{b}=3\hat{j}+4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a}=\hat{i}+\hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1\times\vec{b}_2$ is equal to

(A) $-3\hat{i}+3\hat{j}-9\hat{k}$ (B*) $6\hat{i}+6\hat{j}+\frac{9}{2}\hat{k}$ (C) $-6\hat{i}+6\hat{j}-\frac{9}{2}\hat{k}$ (D) $3\hat{i}-3\hat{j}+9\hat{k}$

Sol. $\vec{b}_1 = \frac{(\vec{b}.\vec{a})\hat{a}}{1}$

$=\left\{\frac{(3\hat{j}+4\hat{k})\cdot(\hat{i}+\hat{j})}{\sqrt{2}}\right\}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

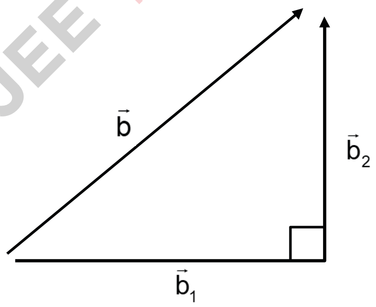
$=\frac{3(\hat{i}+\hat{j})}{\sqrt{2}\times\sqrt{2}}=\frac{3(\hat{i}+\hat{j})}{2}$

$\vec{b}_1+\vec{b}_2=\vec{b}$

$\vec{b}_2=\vec{b}-\vec{b}_1$

$=\left(3\hat{i}+4\hat{k}\right)-\frac{3}{2}\left(\hat{i}+\hat{j}\right)$

$\boxed{\vec{b}_2=-\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+4\hat{k}}$



$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{3}{2} & 4 \end{vmatrix}$

$\hat{i}(6)-\hat{j}(6)+\hat{k}\left(-\frac{9}{4}+\frac{9}{4}\right);\ 6\hat{i}-6\hat{j}+\frac{9}{2}\hat{k}$

25. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is

(A) $\frac{21}{220}$ (B) $\frac{3}{11}$ (C*) $\frac{1}{11}$ (D) $\frac{2}{23}$

26. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then a value of $\frac{P(E)}{P(F)}$ is

(A*) $\frac{4}{3}$ (B) $\frac{3}{2}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$

Sol. $P(\bar{E} \cap \bar{F}) = P(\bar{E}).P(\bar{F}) = \frac{1}{12} = \boxed{xy = \frac{1}{12}}$

$P(\bar{E} \cap \bar{F}) = P(\bar{E}).P(\bar{F}) = \frac{1}{2}$

$\Rightarrow (1 - P(E))(1 - P(F) = \frac{1}{2}$

$= 1 - x - y + xy = \frac{1}{2}$

$1 - x - y + \frac{1}{12} = \frac{1}{2}$

$1 - x - y = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$

$\boxed{x + y = \frac{7}{12}}$

$x + \frac{1}{12x} = \frac{7}{12}$

$\frac{12x^2 + 1}{12x} = \frac{7}{12}$

$12x^2 - 7x + 1 = 0$

$12x^2 - 4x - 3x + 1 = 0$

$4x(3x-1) - 1(3x-1) = 0$

$x = \frac{1}{3}, x = \frac{1}{4}$

$y = \frac{1}{4}, y = \frac{1}{3}$

$\therefore \frac{x}{y} = \frac{1/3}{1/4} = \frac{4}{3} \text{ or } \frac{1/4}{1/3} = \frac{3}{4}$

27. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is

(A) 8.25 (B) 8.50 (C) 8.00 (D*) 9.00

Sol. $\sum_{i=1}^{100} xi = 400$ $\sum_{i=1}^{100} xi^2 = 2475$

variance

$\boxed{\sigma^2 = \frac{\Sigma xi^2}{N} - \left(\frac{\Sigma x_1}{N}\right)^2}$ $3^2 + 4^2 + 5^2$

$9 + 16 + 25 = 50$

$= \frac{2475}{97} - \left(\frac{388}{97}\right)^2$

$\frac{2475}{97} - 16$

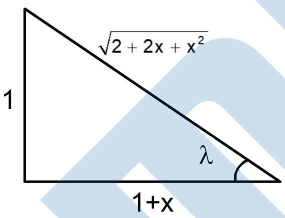
$\frac{2475 - 1552}{97} = \frac{873}{97}$

= (9)

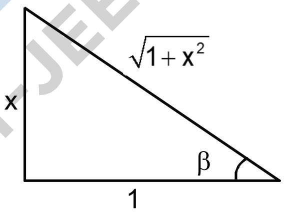
28. A value of x satisfying the equation $\sin[\cot^{-1}(1 + x)] = \cos[\tan^{-1}x]$, is

(A*) $\frac{-1}{2}$ (B) – 1 (C) 0 (D) $\frac{1}{2}$

Sol. $\sin\left[\underset{\lambda}{\underbrace{\cot^{-1}(1+x)}}\right] = \cos\left(\underset{\beta}{\underbrace{\tan^{-1}x}}\right)$



cot λ = 1 + x



$\frac{1}{\sqrt{x^2 + 2x + 2}} = \frac{1}{1\sqrt{1 + x^2}}$

$x^2 + 2x + 2 = x^2 + 1$

$\boxed{x = -1/2}$

29. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is

(A) 12.5 (B) 13.2 (C*) 12 (D) 13

Sol. $\cos 60 = \frac{4 + 25 - c^2}{2.2.5}$

$10 = 29 - c^2$

$c^2 = 19$

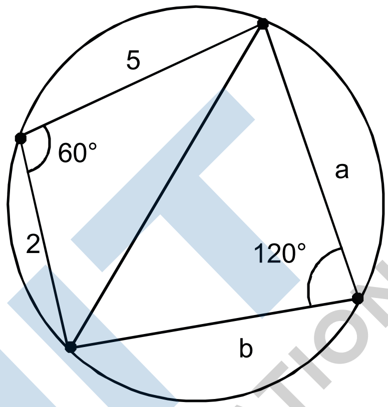
$\boxed{c = \sqrt{19}}$

$-\frac{1}{2} = \frac{a^2 + b^2 - 19}{2ab}$

$a^2 + b^2 - 19 = -ab$

$a^2 + b^2 + ab = 19$

$\text{Area} = \frac{1}{2} \times 2 \times 5 \sin 60 + \frac{1}{2} ab \sin x = 4\sqrt{3}$



$\frac{5\sqrt{3}}{2} + \frac{ab\sqrt{3}}{4} = 4\sqrt{3}$

$\frac{ab}{4} = 4 - \frac{5}{2} = \frac{3}{2}$

$ab = 6$

$a^2+b^2 = 13$

$\boxed{a = 2, b = 3}$

Perimeter

= 2 + 5 + 2+ 3

= 12

30. Contrapositive of the statement

'If two numbers are not equal, then their squares are not equal', is

(A*) If the squares of two numbers are equal, then the numbers are equal.

(B) If the squares of two numbers are equal, then the numbers are not equal.

(C) If the squares of two numbers are not equal, then the numbers are not equal.

(D) If the squares of two numbers are not equal, then the numbers are equal.

Sol. $p \rightarrow q$

contrapositive is

$\sim q \rightarrow \sim p$

PART – B – PHYSICS

31. The electric field component of a monochromatic radiation is given by

$\vec{E} = 2E_0 \hat{i} \cos kz \cos \omega t$

Its magnetic field $\vec{B}$ is then given by :

(A*) $\frac{2E_0}{c} \hat{j} \sin kz \sin \omega t$ (B) $\frac{2E_0}{c} \hat{j} \cos kz \cos \omega t$

(C) $\frac{2E_0}{c} \hat{j} \sin kz \cos \omega t$ (D) $-\frac{2E_0}{c} \hat{j} \sin kz \sin \omega t$

Sol. $\frac{dE}{dz} = -\frac{dE}{dt}$

$\frac{dE}{dz} = -2E_0 K \sin kz \cos \omega t = -\frac{dB}{dt}$

dB = + $2E_0$K sin kz cos ωt dt

B = +$2E_0$K sin kz $\int \cos\omega t dt$

$= +2E_0 \frac{k}{\omega} \sin kz \sin \omega t$

$\frac{E_0}{B_0} = \frac{\omega}{k} = C$

$B = \frac{2E_0}{C} \sin kz \sin \omega t$

32. N moles of a diaotmic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas ?

(A) 0 (B) $\frac{5}{2}$nRT (C) $\frac{3}{2}$nRT (D*) $\frac{1}{2}$nRT

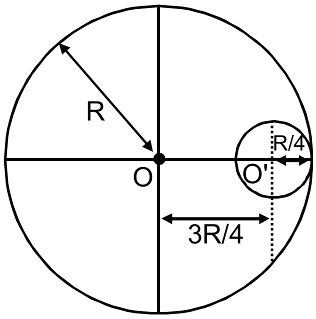
Sol. $u_i = N\frac{5}{2}RT$

$u_f = 2n\frac{3}{2}RT + (N-n)\frac{5}{2}RT$

$= \frac{1}{2}nRT + \frac{5}{2}nRT$

$\Delta u = \frac{1}{2}nRT$

33. A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is -



(A) $\frac{219MR^2}{256}$ (B*) $\frac{237MR^2}{512}$ (C) $\frac{197MR^2}{256}$ (D) $\frac{19MR^2}{512}$

Sol. $I_D = \frac{mr^2}{2}$

$I_{removed} = \frac{1}{2}\frac{m}{16}\frac{r^2}{16} + \frac{m}{16}\frac{9r^2}{16}$ $(Im + md)$

$= \frac{mr^2 + 18mr^2}{512}$

$= \frac{19mr^2}{512}$

$I_{removed} = \frac{mr^2}{2} - \frac{19}{512}mr^2$

$= \frac{237}{512}mr^2$

34. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 Nm^{-1} and oscillates in a damping medium of damping constant 10^{-2} kg s^{-1}. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to -

(A) 2 s (B) 5 s (C) 7 s (D*) 3.5 s

Sol. $E' = \frac{1}{2}$ $b = \frac{\lambda}{m}$ $A = A_0 e^{-bt}$

$a' = \frac{a}{\sqrt{2}}$ $\lambda = bm$ $\frac{1}{\sqrt{2}} = e^{-\frac{t}{10}}$

$b = \frac{1}{10}$ $\sqrt{2} = e^{\frac{1}{10}}$

$\ln\sqrt{2} = \frac{t}{10}$

t = 3.5

35. A steel rail of length 5 m and area of cross section 40 cm^2 is prevented from expanding along its length while the temperature rises by 10°C. If coefficient of linear expansion and Young's modulus of steel are 1.2×10^{-5} K^{-1} and 2×10^{11} Nm^{-2} respectively, the force developed in the rail is approximately :

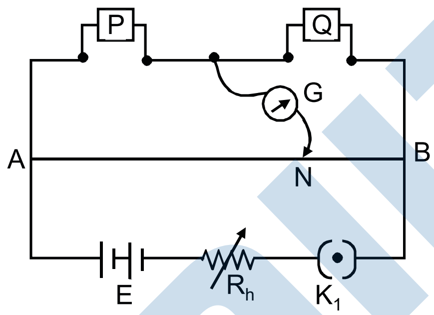
(A) 2×10^7 N (B) 2×10^9 N (C) 3×10^{-5} N (D*) 1×10^5 N

Sol. $F = yA \propto \Delta t$

$= 2 \times 10^{11} \times 40 \times 10^{-4} \times 1.2 \times 10^{-5} \times 10$

$= 9.6 \times 10^4 = 1 \times 10^5$ N

36. In a meter bridge experiment resistances are connected as shown in the figure. Initially resistance P = 4 Ω and the neutral point N is at 60 cm from A. Now an unknown resistance R is connected in series to P and the new position of the neutral point is at 80 cm from A. The value of unknown resistance R is -



(A) $\frac{33}{5}\Omega$ (B) $6\,\Omega$ (C*) $\frac{20}{3}\Omega$ (D) $7\,\Omega$

Sol. Initially $\frac{4}{60} = \frac{Q}{40}$ $Q = \frac{16}{6} = \frac{8}{3}\Omega$

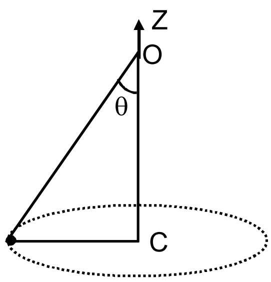
Finally $\frac{4+R}{80} = \frac{Q}{20} = \frac{8}{60}$

$4 + R = \frac{64}{6}$

$R = \frac{64}{6} - 4 = \frac{64-24}{6} = \frac{40}{6}$

$= \frac{20}{3}\Omega$

37. A conical pendulum of length 1 m makes an angle θ = 45° w.r.t. Z-axis and mvoes in a circle in the XY plane. The radius of the circle is 0.4 m and its center is vertically below O. The speed of the pendulum, in its circular path, will be - (Take g = 10 ms–2)



(A) 0.2 m/s (B) 0.4 m/s (C*) 2 m/s (D) 4 m/s

Sol. $T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

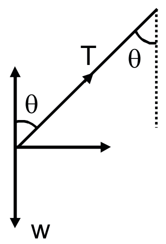
$\tan\theta = \frac{v^2}{rg}$

$\theta = 45°$ w

$v^2 = rg$

$v = \sqrt{rg} = \sqrt{0.4 \times 10}$

= 2 m/s



38. A signal is to be transmitted through a wave of wavelength λ, using a linear antenna. The length ℓ of the antenna and effective power radiated P_{eff} will be given respectively as -

(K is a constant of proportionality)

(A) $\frac{\lambda}{8}, P_{eff} = K\left(\frac{1}{\lambda}\right)$ (B) $\frac{\lambda}{16}, P_{eff} = K\left(\frac{1}{\lambda}\right)^3$ (C) $\frac{\lambda}{5}, P_{eff} = K\left(\frac{1}{\lambda}\right)^{\frac{1}{2}}$ (D*) $\lambda, P_{eff} = K\left(\frac{1}{\lambda}\right)^2$

Sol. Length of antenna = comparable to λ

Power $P = \mu\left(\frac{1}{\lambda}\right)^2$

here μ = k

39. A sinusoidal voltage of peak value 283 V and angular frequency 320/s is applied to a series LCR circuit. Given that R = 5 Ω, L = 25 mH and C = 1000 μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be -

(A) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (B*) 7 Ω and 45°

(C) 7 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (D) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$

Sol. e_0 = 283 volt ω = 320

$x_L = 320 \times 25 \times 10^{-3} = 8\,\Omega$

$x_C = \frac{1}{\omega c} = \frac{1}{320 \times 1000 \times 10^{-6}}$

$= \frac{1000}{320} = 3.1\,\Omega$

R = 5 Ω

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50} = 7\ \Omega$

$\tan\phi = \frac{X_L - x_C}{R}$

= 1 ϕ = 45°

40. A single slit of width 0.1 mm is illuminated by a parallel beam of light of wavelength 6000 Å and diffraction bands are observed on a screen 0.5 m from the slit. The distance of the third dark band from the central bright band is -

(A*) 9 mm (B) 3 mm (C) 4.5 mm (D) 1.5 mm

Sol. $a = 0.1 \text{ mm} = 10^{-4}$

$\lambda = 6000 \times 10^{-10} = 6 \times 10^{-7}$

D = 0.5 m

for 3rd dark

$a \sin\theta = 3\lambda$

$$\sin\theta = \frac{3\lambda}{a} = \frac{x}{D}$$

$$x = \frac{3\lambda D}{a}$$

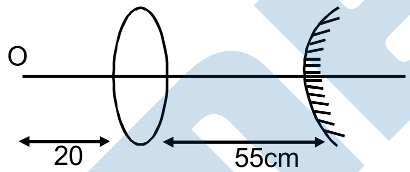
$$= \frac{3 \times 6 \times 10^{-7} \times 0.5}{10^{-4}}$$

= 9 mm

41. In an experiment a convex lens of focal length 15 cm is placed coaxially on an optical bench in front of a convex mirror at a distance of 5 cm from it. It is found that an object and its image coincide, if the object is placed at a distance of 20 cm from the lens. The focal length of the convex mirror is -

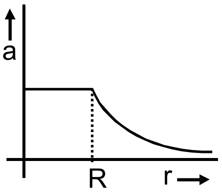
(A) 20.0 cm (B) 30.5 cm (C) 25.0 cm (D*) 27.5 cm

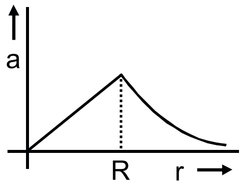
Sol.

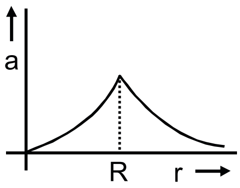


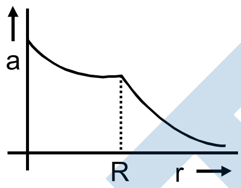
due to this image will from 55 cm behind convex mirror then reflection takes replace due to mirror image will form at v behind mirror (let) this will here as object for lens and final image will from at a distance of 20 cm from lens (i.e. obext) if focal length in of convex lens $= \frac{55}{2}$ cm

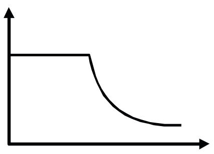
42. The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \leq R$ and $\rho(r) = 0$ for $r > R$, Where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a, of a test particle as a function of r is -

(A*) 

(B) 

(C) 

(D) 

Sol. 

$\frac{M}{V} = \frac{k}{r}$ for inside

$M = \frac{kv}{r}$

$g = \frac{Gmr}{R^3} = \frac{G}{R^3} \cdot \frac{kv}{r} \cdot r = \text{constant}$

$g_{out} = \frac{GM}{R^2}$

43. A uniform wire of length I and radius r has a resistance of 100 Ω. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be -

(A*) 1600 Ω (B) 100 Ω (C) 200 Ω (D) 400 Ω

Sol. $R = \frac{\rho l}{A}$ $\quad Al = V$

$R = \frac{\rho v}{A^2}$

$\rho \to$ constant

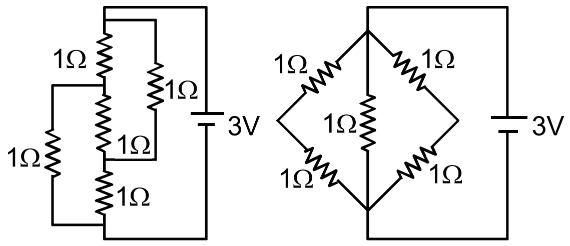
$v \to$ constant

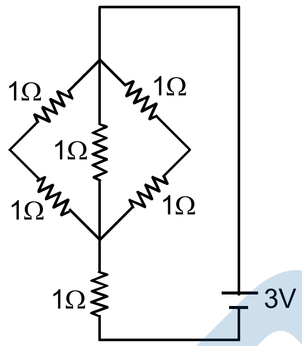
$R \propto \frac{1}{A^2} \propto \frac{1}{r^2}$

$R \propto \frac{1}{r^4}$

$R_2 = 16\ R_1 = 1600\ \Omega$

44. The figure shows three circuits I, II and III which are connected to a 3V battery. If the powers dissipated by the configurations I, II and III are P_1, P_2 and P_3 respectively, then -





(A*) $P_2 > P_1 > P_3$ (B) $P_1 > P_2 > P_3$ (C) $P_3 > P_2 > P_1$ (D) $P_1 > P_3 > P_2$

Sol. $P = \frac{V^2}{R}$

$P \propto \frac{1}{R}$

$R_1 = 1$

$R_2 = 1/2$

$R_3 = 2$

$P_2 > P_1 > R_3$

45. A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

$$y(x,t) = 0.5\sin\left(\frac{5\pi}{4}x\right)\cos(200\ \pi t)$$

What is the speed of the travelling wave moving in the positive x direction ?

(x and t are in meter and second, respectively.)

(A) 120 m/s (B) 90 m/s (C*) 160 m/s (D) 180 m/s

Sol. $V = \frac{w}{k}$

$= \frac{200\pi}{5\pi/4} = 160$

46. In an experiment to determine the period of a simple pendulum of length 1m, it is attached to different spherical bobs of radii r_1 and r_2. The two spherical bobs have uniform mass distribution. If the relative difference in the periods, is found to be 5×10^{-4} s, the difference in radii, $|r_1 - r_2|$ is best given by -

(A) 0.01 cm (B*) 0.1 cm (C) 0.5 cm (D) 1 cm

Sol. $T \propto \sqrt{l}$ $l = 1$

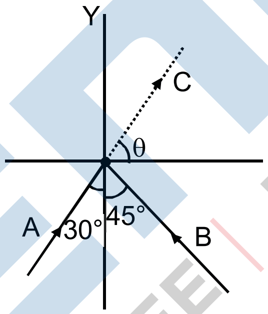
$$\frac{\Delta T}{l} = \frac{1}{2}\frac{\Delta l}{l} \qquad \Delta l = r_1 - r_2$$

$$5 \times 10^{-4} = \frac{1}{2}\frac{r_1 - r_2}{1}$$

$r_1 - r_2 = 10 \times 10^{-4}$

10^{-3} m = 10^{-1} cm = 0.1 cm

47. Two particles A and B of equal mass M are moving with the same speed υ as shown in the figure. They collide completely inelastically and mvoe as a single particle C. The angle θ that the path of C makes with the X-axis is given by -



(A) $\tan\theta = \frac{\sqrt{3} - \sqrt{2}}{1 - \sqrt{2}}$ (B) $\tan\theta = \frac{1 - \sqrt{2}}{\sqrt{2}(1 + \sqrt{3})}$

(C) $\tan\theta = \frac{1 - \sqrt{3}}{1 + \sqrt{2}}$ (D*) $\tan\theta = \frac{\sqrt{3} + \sqrt{2}}{1 - \sqrt{2}}$

Sol. $2mv' \sin\theta = \frac{mv}{\sqrt{2}} + \frac{mv\sqrt{3}}{2}$

$$3mv' \cos\theta = \frac{mv}{2} - \frac{mv}{\sqrt{2}}$$

$$\sin\theta = \frac{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2} + \sqrt{3}}{1 - \sqrt{2}}$$

48. A laser light of wavelength 660 nm is used to weld Retina detachment. If a laser pulse of width 60 ms and power 0.5 kW is used the approximate number of photons in the pulse are -

[Take Planck's constant h = 6.62 × 10^{-34} Js]

(A) 10^{22} (B) 10^{19} (C*) 10^{20} (D) 10^{18}

Sol. $P = \frac{nhc}{\lambda t}$ $n = \frac{p\lambda t}{hc}$

$= 5 \times 10^3 \times \frac{660 \times 10^{-9} \times 60 \times 10^{-3}}{6.6 \times 10^{-34} \times 3 \times 10^8}$

= 100 × 10^{18} = 10^{20}

49. The current gain of a common emitter amplifier is 69. If the emitter current is 7.0 mA, collector current is-

(A) 69 mA (B) 0.69 mA (C*) 6.9 mA (D) 9.6 mA

Sol. $\beta = 69 = \frac{I_C}{I_B}$

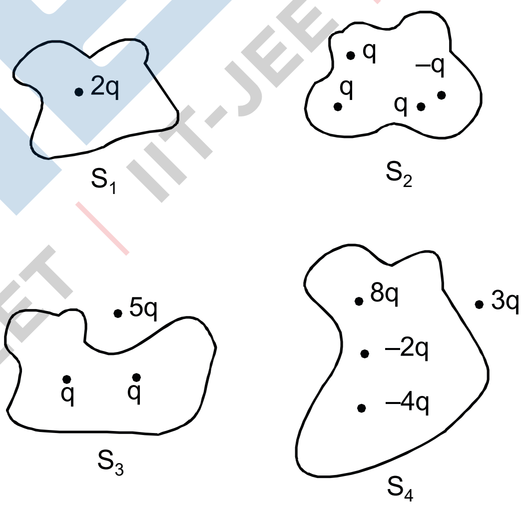
$\alpha = \frac{\beta}{1+\beta} = \frac{69}{70} = \frac{I_C}{I_E}$

$I_C = I_E \times \frac{69}{70}$

$= \frac{69}{70} \times 7$

= 6.9 Ma

50. Four closed surfaces and corresponding charge distributions are shown below –



Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3 and Φ_4. Then -

(A) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$ (B) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$

(C) $\Phi_1 > \Phi_3$; $\Phi_2 > \Phi_4$ (D*) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$

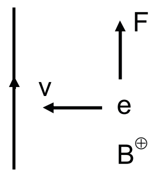
Sol. Eq = same in all Φ = Same

51. A negative test charge is moving near a long straight wire carrying a current. The force acting on the test charge is parallel to the direction of the current. The motion of the charge is -

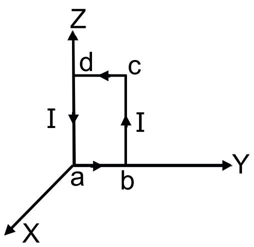
(A) away from the wire (B*) towards the wire

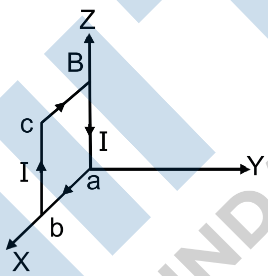
(C) parallel to the wire along the current (D) parallel to the wire opposite to the current

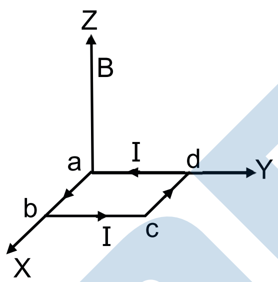
Sol. $\vec{F} = q(\vec{v} \times \vec{B})$

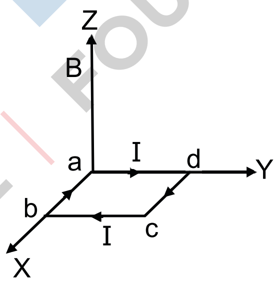


52. A uniform magnetic field B of 0.3 T is along the positive Z-direction. A rectangular loop (abcd) of sides 10 cm × 5 cm carries a current I of 12 A. Out of the following different orientations which one corresponds to stable equilibrium ?

(A) 

(B) 

(C*) 

(D) 

Sol. M = NIA

$\vec{M}$ and $\vec{B}$ are on same direction

53. The acceleration of an electron in the first orbit of the hydrogen atom (n = 1) is -

(A) $\frac{h^2}{4\pi m^2 r^3}$ (B) $\frac{h^2}{\pi^2 m^2 r^3}$ (C) $\frac{h^2}{8\pi^2 m^2 r^3}$ (D*) $\frac{h^2}{4\pi^2 m^2 r^3}$

Sol. $v = \frac{I^2}{2h\varepsilon_0}$

$r = \frac{h^2\varepsilon_0}{\pi m e^2} \Rightarrow \varepsilon_0 = \frac{r\pi m e^2}{h^2}$

$\frac{v^2}{r} = \frac{I^6 \pi m}{4h^4\varepsilon_0^3} = \frac{h^2}{4r^3\pi^2 m^2}$

54. Two tubes of radii r_1 and r_2, and lengths l_1 and l_2, respectively, are connected in series and a liquid flows through each of them in stream line conditions. P_1 and P_2 are pressure differences across the two tubes. If P_2 is $4P_1$ and l_2 is $\frac{l_1}{4}$, then the radius r_2 will be equal to -

(A) $4r_1$ (B) r_1 (C) $2r_1$ (D*) $\frac{r_1}{2}$

Sol. $\frac{\phi v}{dt} = \frac{\pi}{8}\frac{pr^4}{nL}$

$\frac{p_1 r_1^4}{L_1} = \frac{p_2 r_2^4}{L_2}$

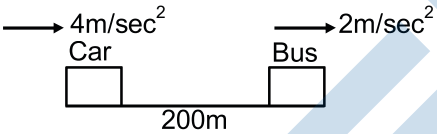
$\frac{p_1 r_1^4}{l_1} = \frac{4p_2 r_2^4}{l_{1/4}} = r_2^4 = \frac{r_1^4}{16}$

$r_2 = \frac{r_1}{2}$

55. A car is standing 200 m behind a bus, which is also at rest. The two-start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after at time of -

(A) 120 s (B) 15 s (C) 110 s (D*) 10 2 s

Sol.

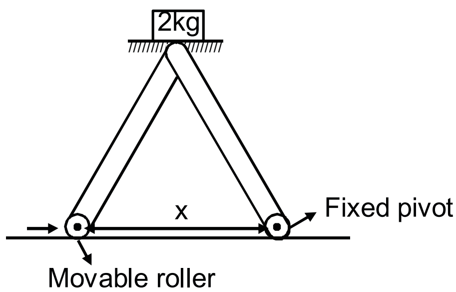


$a_{CB} = 2$ m/sec^2

$200 = \frac{1}{2} \times 2t^2$

$t = 10\sqrt{2}$ second

56. The machine as shown has 2 rods of length 1 m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot. As the roller goes back and forth, a 2 kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a -



(A) speed which is $\frac{3}{4}$ th of that of the roller when the weight is 0.4 m above the ground

(B) constant speed

(C*) decreasing speed

(D) increasing speed

57. A physical quantity P is described by the relation

$P = a^{1/2} b^2 c^3 d^{-4}$

If the relative errors in the measurement of a,b,c and d respectively, are 2%, 1%, 3% and 5%, then the relative error in P will be -

(A) 12% (B) 8% (C) 25% (D*) 32%

Sol. $\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c} + 4\frac{\Delta d}{d}$

$= \frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5$

= 32%

58. Imagine that a reactor converts all given mass into energy and that it operates at a power level of 109 watt. The mass of the fuel consumed per hour in the reactor will be :

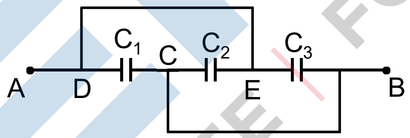
(velocity of light, c is 3 × 108 m/s)

(A) 6.6×10^{-5} gm (B) 0.96 gm (C*) 4×10^{-2} gm (D) 0.8 gm

Sol. $P = \frac{E}{\Delta t} = \frac{\Delta mc^2}{\Delta t}$

$\frac{\Delta m}{\Delta t} = \frac{P}{c^2} = \frac{10^9}{(3 \times 10^8)^2} = 4 \times 10^{-2}\text{gm}$

59. A combination of parallel plate capacitors is maintained at a certain potential difference.



When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.

(A) 6 (B) 4 (C) 3 (D*) 5

Sol. $C_1 = \frac{\varepsilon_0 A}{3}$ before

$C_1 = \frac{k\varepsilon_0 A}{3} + \frac{\varepsilon_0 A}{2.4}$ after

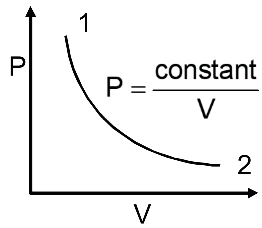
$\frac{\varepsilon_0 A}{3} = \frac{k\frac{\varepsilon_0 A}{3} \cdot \frac{\varepsilon_0 A}{2.4}}{k\frac{\varepsilon_0 A}{3} + \frac{\varepsilon_0 A}{2.4}}$

3k = 2.4k + 3

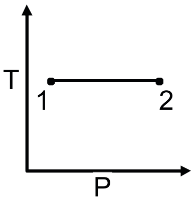
$0.6k = 3 \Rightarrow k = \frac{3}{0.6}$

$k = \frac{30}{6} = 5$

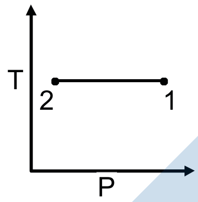
60. For the P-V diagram given for an ideal gas, out of the following which one correctly represents the T-P diagram ?



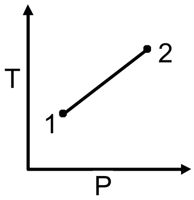
(A)



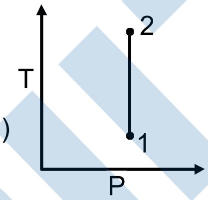
(B*)



(C)



(D)



Sol. $P \propto \frac{1}{V}$ $T = \text{constant}$

PART – C – CHEMISTRY

61. An ideal gas undergoes isothermal expansion at constant pressure. During the process:

(A) enthalpy increases but entropy decreases.

(B*) enthalpy remains constant but entropy increases.

(C) enthalpy decreases but entropy increases.

(D) Both enthalpy and entropy remain constant.

Sol. $\Delta H = nC_p\Delta T = 0$

$\Delta S = nR\ln(V_f/V_i) \geq 0$

62. 50 mL of 0.2 M ammonia solution is treated with 25 mL of 0.2 M HCl. If pK_b of ammonia solution is 4.75, the pH of the mixture will be:

(A) 3.75 (B*) 4.75 (C) 8.25 (D) 9.25

Sol. $NH_3 \quad + \quad HCl \longrightarrow \quad NH_4Cl$

$\frac{50 \times 0.2}{1000} \qquad \frac{25 \times 0.2}{1000}$

5 0 5

Buffer solution

$$\text{pOH} = \text{pkb NH}_3 + \text{Hg}\frac{\text{salt}}{\text{base}} = 4.75$$

63. The electron in the hydrogen atom undergoes transition from higher orbitals to orbital of radius 211.6 pm. This transition is associated with:

(A) Lyman series (B*) Balmer series (C) Paschen series (D) Brackett series

Sol. R = 211.6 pm = 2.11 Å

$R = 0.529 \times \frac{n^2}{Z} = 2.11\ \text{Å}$ $\qquad n^2 = 4 \Rightarrow n = 2$

64. At 300 K, the density of a certain gaseous molecule at 2 bar is double to that of dinitrogen (N_2) at 4 bar. The molar mass of gaseous molecule is:

(A*) 28 g mol^{-1} (B) 56 g mol^{-1} (C) 112 g mol^{-1} (D) 224 g mol^{-1}

65. What quantity (in mL) of a 45% acid solution of a mono-protic strong acid must be mixed with a 20% solution of the same acid to produce 800 mL of a 29.875 % acid solution?

(A) 320 (B) 325 (C*) 316 (D) 330

Sol. $$\frac{V \times 45}{100} + \frac{(800 - V)20}{100} = \frac{800 \times 29.875}{100}$$

$$\frac{9V}{20} + 160 - \frac{V}{5} = 239$$

$$\frac{5V}{20} = 79 \Rightarrow V = 316 \text{ Ans.}$$

66. To find the standard potential of M^{3+}/M electrode, the following cell is constituted:

$Pt / M / M^{3+}(0.001\ mol\ L^{-1}) / Ag^{+} (0.01\ mol\ L^{-1}) / Ag$

The emf of the cell is found to be 0.421volt at 298 K. The standard potential of half reaction

$M^{3+} + 3e^{-} \rightarrow M$ at 298 K will be:

[Given: $E^{-}_{Ag^{+}/Ag}$ at 298 K = 0.80 Volt]

(A) 0.38 Volt (B*) 0.32 Volt (C) 1.28 Volt (D) 0.66 Volt

Sol. $0.421 = E^{\circ} - \frac{0.059}{3}\log\frac{0.001}{(0.01)^3}$

$E^{\circ} = 0.421 + \frac{0.059}{3}\log(10^3)$

$E^{\circ} = 0.480 = 0.8 - E^{\circ}_{M^{+3}/M}$

$E^{\circ}_{M^{+3}/M} = 0.32$

67. A gas undergoes change from state A to state B. In this process, the heat absorbed and work done by the gas is 5 J and 8 J, respectively. Now gas is brought back to A by another process during which 3 J of heat is evolved. In this reverse process of B to A:

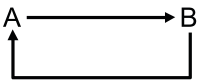
(A) 10 J of the work will be done by the gas.

(B) 6 J of the work will be done by the gas.

(C) 10 J of the work will be done by the surrounding on gas.

(D*) 6 J of the work will be done by the surrounding on gas.

Sol. q = +5 W = – 8 J, ΔU_{AB} = –3



q = –3 ΔU_{BA} = +3

W_{BA} = 6J

68. Adsorption of a gas on a surface follows Freundlich adsorption isotherm. Plot of log $\frac{x}{m}$ versus log p gives a straight line with slope equal to 0.5, then:

($\frac{x}{m}$ is the mass of the gas adsorbed per gram of adsorbent)

(A) Adsorption is independent of pressure.

(B) Adsorption is proportional to the pressure.

(C*) Adsorption is proportional to the square root of pressure.

(D) Adsorption is proportional to the square of pressure.

Sol. $\log\left(\frac{x}{m}\right) = \frac{1}{2}\log(P) + K$

$\frac{x}{m} = KP^{1/2}$

69. The rate of a reaction quadruples when the temperature changes from 300 to 310 K. The activation energy of this reaction is:

(Assume activation energy and preexponential factor are independent of temperature; ln 2=0.693; R=8.314 J mol^{-1} K^{-1})

(A*) 107.2 kJ mol^{-1} (B) 53.6 kJ mol^{-1} (C) 26.8 kJ mol^{-1} (D) 214.4 kJ mol^{-1}

Sol. $4 = e^{\frac{Ea}{R}\left(\frac{1}{300}-\frac{1}{310}\right)}$

$$\ln(4) = \frac{Ea}{R}\left\{\frac{10}{300\times 310}\right\}$$

$$Ea = \frac{0.693\times 2\times 8.314\times 300\times 310}{10}$$

= 107165.79 J = 107.165 KJ

70. A solution is prepared by mixing 8.5 g of CH_2Cl_2 and 11.95 g of $CHCl_3$. If vapour pressure of CH_2Cl_2 and $CHCl_3$ at 298 K are 415 and 200 mmHg respectively, the mole fraction of $CHCl_3$ in vapour form is: (Molar mass of Cl=35.5 g mol^{-1})

(A) 0.162 (B) 0.675 (C*) 0.325 (D) 0.486

71. The electronic configuration with the highest ionization enthalpy is:

(A) [Ne] $3s^2$ $3p^1$ (B) [Ne] $3s^2$ $3p^2$ (C*) [Ne] $3s^2$ $3p^3$ (D) [Ar] $3d^{10}$ $4s^2$ $4p^3$

Sol. $S_1 < P_1 < S_2 < P_2 < P_4 < P_3 < P_5 < P_6$ (IE order)

72. The following reaction occurs in the Blast Furnace where iron ore is reduced to iron metal:

$Fe_2O_3(s)+3\ CO(g) \rightleftharpoons 2\ Fe(l) + 3\ CO_2(g)$

Using the Le Chatelier's principle, predict which one of the following will not disturb the equilibrium?

(A) Removal of CO (B) Removal of CO_2 (C) Addition of CO_2 (D*) Addition of Fe_2O_3

73. Which one of the following is an oxide?

(A) KO_2 (B) BaO_2 (C*) SiO_2 (D) CsO_2

Sol. SiO_2

74. Which of the following is a set of green house gases?

(A) CH_4, O_3, N_2, SO_2 (B) O_3, N_2, CO_2, NO_2

(C) O_3, NO_2, SO_2, Cl_2 (D*) CO_2, CH_4, N_2O, O_3

75. The group having triangular planar structures is:

(A) BF_3, NF_3, CO_3^{2-} (B*) CO_3^{2-}, NO_3^-, SO_3

(C) NH_3, SO_3, CO_3^{2-} (D) NCl_3, BCl_3, SO_3

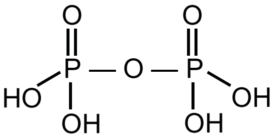
Sol. CO_3^{2-}, NO_3^-, SO_3 : SP^2 Hybridised

76. XeF_6 on partial hydrolysis with water produces a compound 'X'. The same compound 'X' is formed when XeF_6 reacts with silica. The compound 'X' is:

(A) XeF_2 (B) XeF_4 (C*) $XeOF_4$ (D) XeO_3

77. The number of P – OH bonds and the oxidation state of phosphorus atom in pyrophosphoric acid ($H_4P_2O_7$) respectively are:

(A) four and four (B) five and four (C) five and five (D*) four and five

Sol. 

78. Which of the following ions does not liberate hydrogen gas on reaction with dilute acids?

(A) Ti^{2+} (B) V^{2+} (C) Cr^{2+} (D*) Mn^{2+}

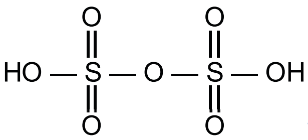
Sol. Mn^{2+}

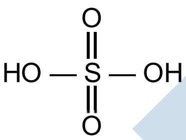
79. The correct sequence of decreasing number of π-bonds in the structures of H_2SO_3, H_2SO_4 and $H_2S_2O_7$ is:

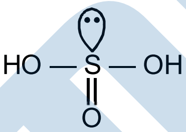
(A) $H_2SO_3 > H_2SO_4 > H_2S_2O_7$ (B) $H_2SO_4 > H_2S_2O_7 > H_2SO_3$

(C) $H_2S_2O_7 > H_2SO_4 > H_2SO_3$ (D*) $H_2S_2O_7 > H_2SO_3 > H_2SO_4$

Sol.

$H_2S_2O_7$ = 

H_2SO_4 = 

H_2SO_3 = 

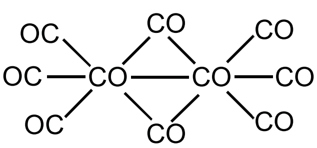
80. $[Co_2(CO)_8]$ displays:

(A*) one Co-Co bond, six terminal CO and two bridging CO

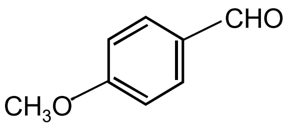
(B) one Co-Co bond, four terminal CO and four bridging CO

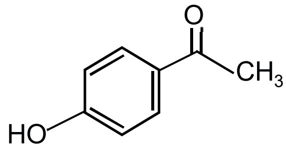
(C) no Co-Co bond, six terminal CO and two bridging CO

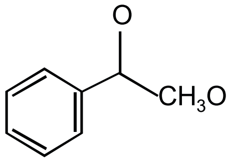
(D) no Co-Co bond, four terminal CO and four bridging CO

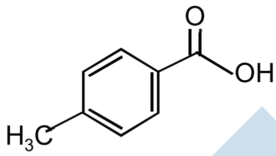
Sol. 

81. A compound of molecular formula $C_8H_8O_2$ reacts with acetophenone to form a single cross-aldol product in the presence of base. The same compound on reaction with conc. NaOH forms benzyl alcohol as one of the products. The structure of the compound is:

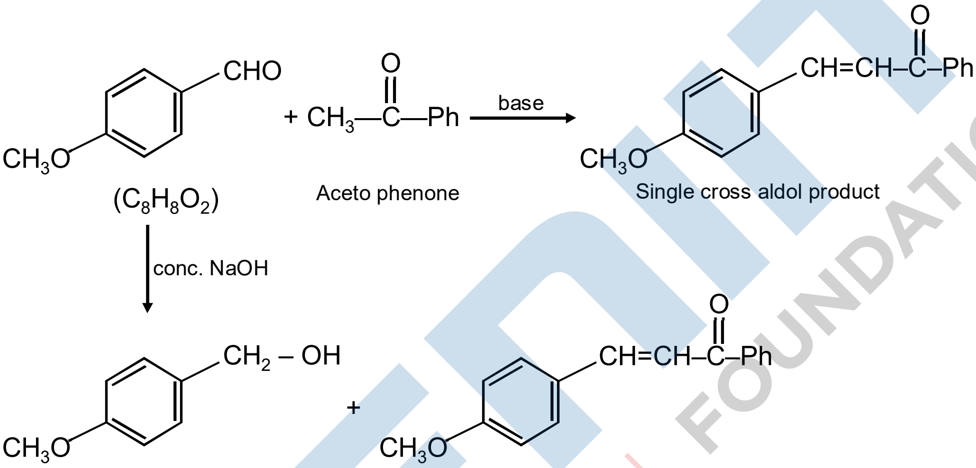
(A*) 

(B) 

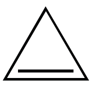
(C) 

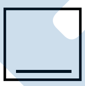
(D) 

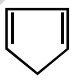
Sol.



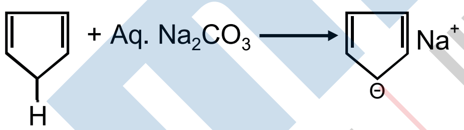
82. Which of the following compounds is most reactive to an aqueous solution of sodium carbonate?

(A) 

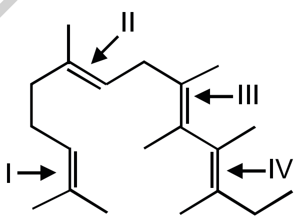
(B) 

(C*) 

(D) 

Sol. 

83. In the following structure, the double bonds are marked as I, II, III and IV

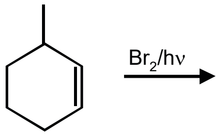


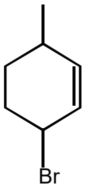
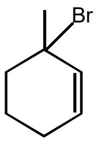
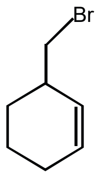
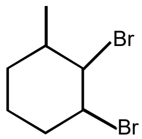
Geometrical isomerism is not possible at site (s):

(A) III (B*) I (C) I and III (D) III and IV

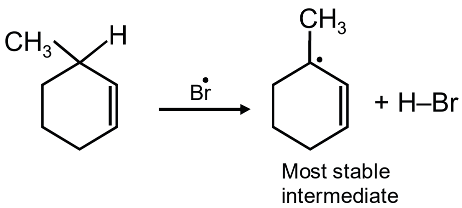
Sol. Different group should be attached to each sp^2 hybridised c-atom.

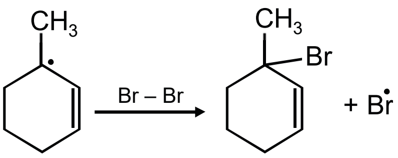
84. The major product of the following reaction is:



(A)  (B*)  (C)  (D) 

Sol. $Br_2 \xrightarrow{h\nu} 2\dot{Br}$





85. The incorrect statement among the following is:

(A) α-D-glucose and β-D-glucose are anomers.

(B*) α-D-glucose and β-D-glucose are enantiomers.

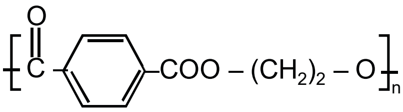
(C) Cellulose is a straight chain polysaccharide made up of only β-D-glucose units.

(D) The penta acetate of glucose does not react with hydroxyl amine.

Sol. α-D-Glucose and β-D-glucose are anomer not enantiomer.

86. Which of the following is a biodegradable polymer?

(A*) $\left[HN - (CH_2)_5CONH - CH_2 - \overset{O}{\overset{\|}{C}} \right]_n$ (B) $\left[HN - (CH_2)_5 - \overset{O}{\overset{\|}{C}} \right]_n$

(C) $\left[HN - (CH_2)_6NHCO - (CH_2)_4 - \overset{O}{\overset{\|}{C}} \right]_n$ (D) 

Sol. $\left[HN - (CH_2)_5CONH - CH_2 - \overset{O}{\overset{\|}{C}} \right]_n$

Nylon–2–Nylon–6

Polymer of glycine and amino caproic acid

87. The increasing order of the boiling points for the following compounds is:

(I) C_2H_5OH (II) C_2H_5Cl (III) $C_2H_5CH_3$ (IV) $C_2H_5OCH_3$

(A*) (III) < (IV) < (II) < (I) (B) (IV) < (III) < (I) < (II)

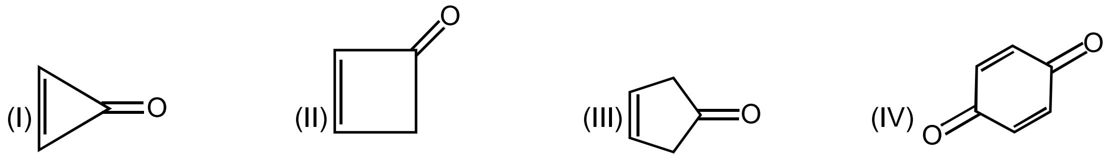
(C) (II) < (III) < (IV) < (I) (D) (III) < (II) < (I) < (IV)

Sol. B.P. $\propto$ dipole moment

$\propto$ H-bonding

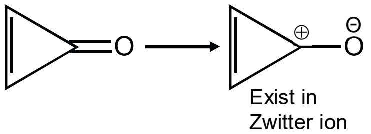
$C_2H_5CH_3 < C_2H_5OCH_3 < C_2H_5Cl < C_2H_5OH$

88. Which of the following compounds will show highest dipole moment?



(A) (I) (B*) (II) (C) (III) (D) (IV)

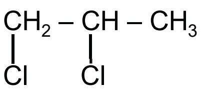
Sol.

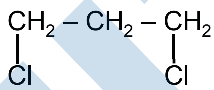


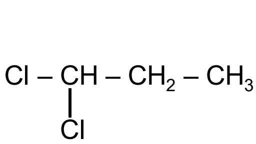
89. In the following reaction sequence:

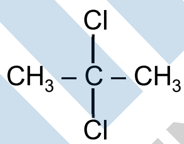
I ($C_3H_6Cl_2$) $\xrightarrow{\text{KOH(aq.)}}$ II $\xrightarrow[\text{(ii) } H_2O/H^+]{\text{(i) } CH_3MgBr}$ III $\xrightarrow{\text{anhy.}ZnCl_2 \text{ +conc.HCl}}$ Gives turbidity immediately

The compound I is:

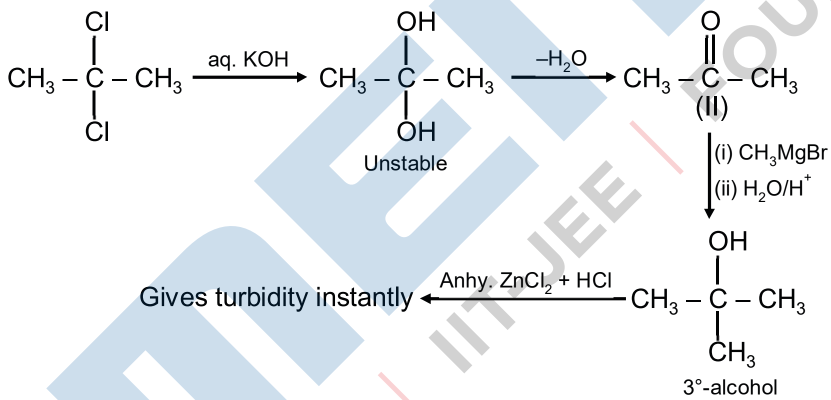
(A) 

(B) 

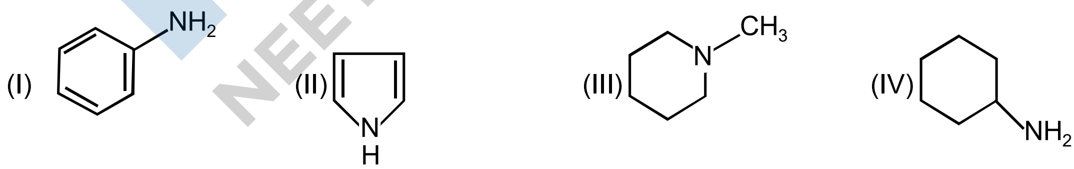
(C) 

(D*) 

Sol.



90. Among the following compounds, the increasing order of their basic strength is:



(A) (I) < (II) < (IV) < (III) (B) (I) < (II) < (III) < (IV)

(C) (II) < (I) < (IV) < (III) (D*) (II) < (I) < (III) < (IV)

Sol. Order of basicity